Revisit rainbow physics through ray refraction by an acrylic sphere

Shu-Ya Chen(陳書亞), Wei-Ting Kao(高瑋婷)，Pi-Hui Tuan(段必輝)

Department of Physics, National Chung Cheng University

Abstract

The outgoing ray characteristics corresponding to quasi-monochromatic beams of red, green, and blue colors parallelly incident into a transparent acrylic sphere are studied to demonstrate the underlying physics of rainbow formation. By the measured dependence of viewing angle α on the impact parameter u that is highly consistent with the theoretical prediction, the color distribution of rainbow can be explicitly resolved.

Overview

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03 Objectives
04 Theoretical Principle
05 Experimental setup
06 Result
07 Conclusion

Motivation

In daily life, the colors of a rainbow are observed at different viewing angles.

Problem

01 Why a rainbow splits into multiple colors and what determines the order of their arrangement.

02 Why each color appears at a fixed elevation angle.

03 How these phenomena are related to the refractive index.

04 Is the angle at which the rainbow appears related to the light intensity?

Objectives

Objective 1

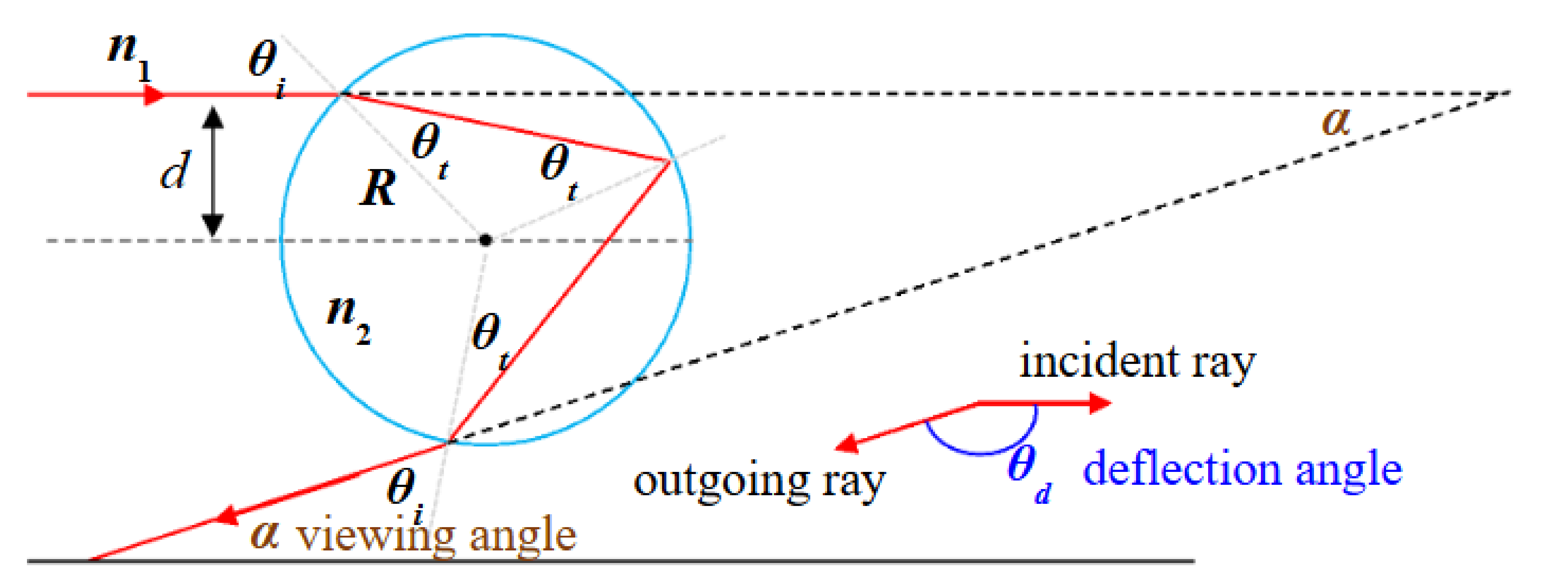
To understand the maximum viewing angle of the rainbow formed through refraction by the acrylic sphere.

Objective 2

Understand the relationship between light intensity and the elevation angle, and explore whether there is any correlation.

Theoretical Principle

The angle between the outgoing refracted ray and the ground corresponds to the viewing angle α of rainbow seen by an observer.



Maximum viewing angle

$$\left.\frac{d\alpha}{du}\right|_{u_c} = 0$$

$$u_c = \sqrt{\frac{4-n^2}{3}}$$

$$\alpha_{max} = 4\sin^{-1}\left(\frac{u_c}{n}\right) - 2\sin^{-1}(u_c)$$

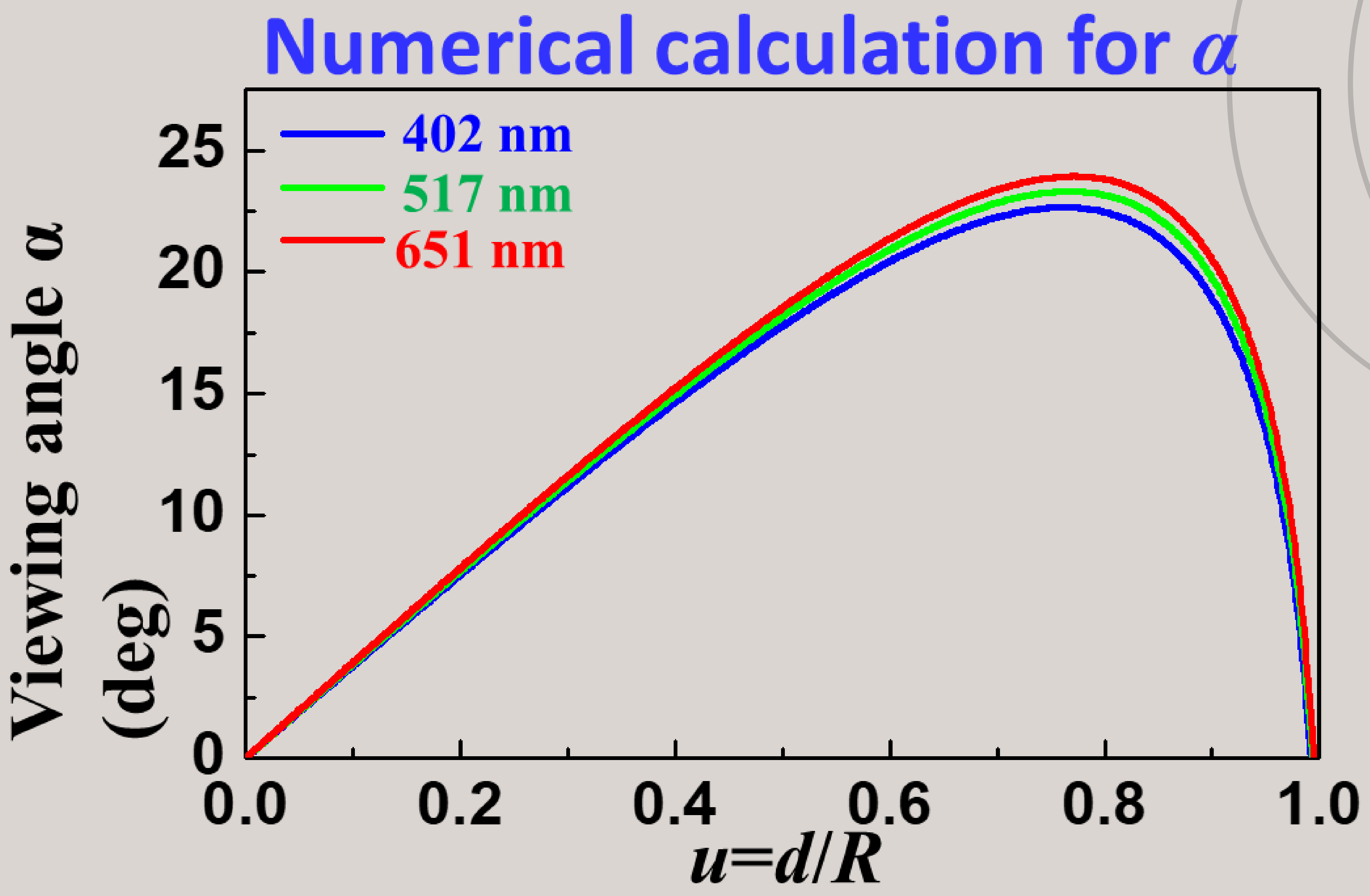
where $u=\sin\theta_i = d/R$ and $n=n_2/n_1$.

From above geometry and Snell's law:

$$\theta_d = 180° - \alpha$$

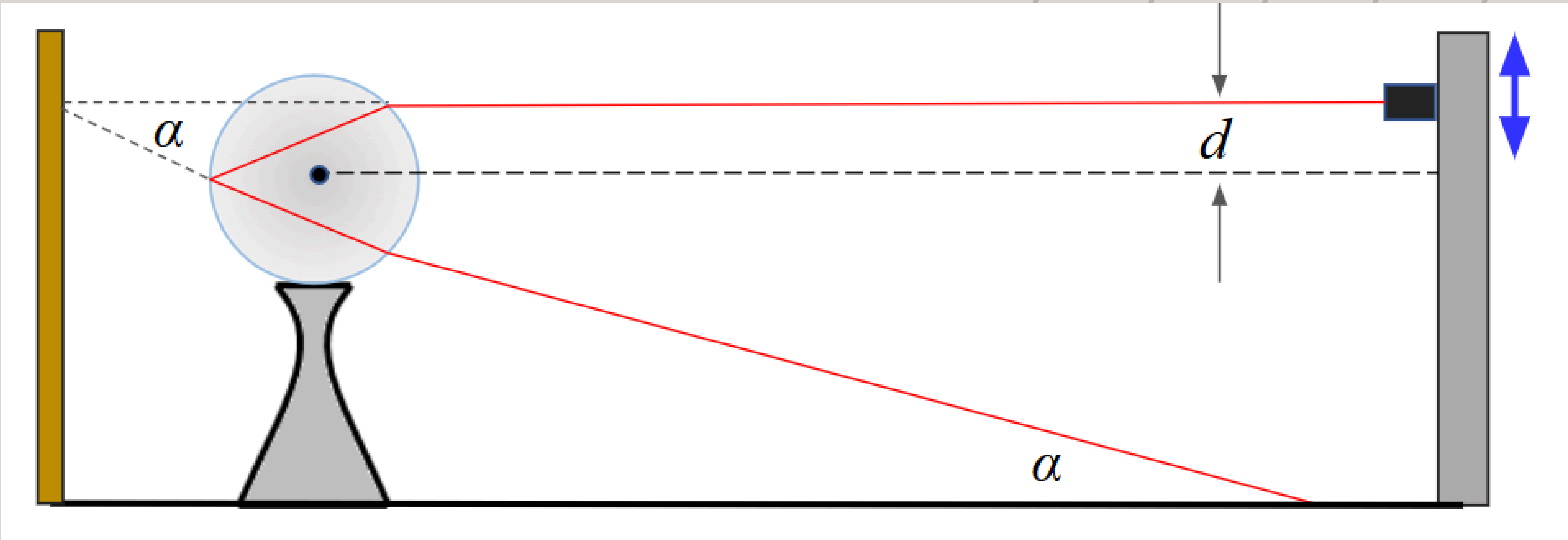
$$\alpha = 4\theta_t - 2\theta_i.$$

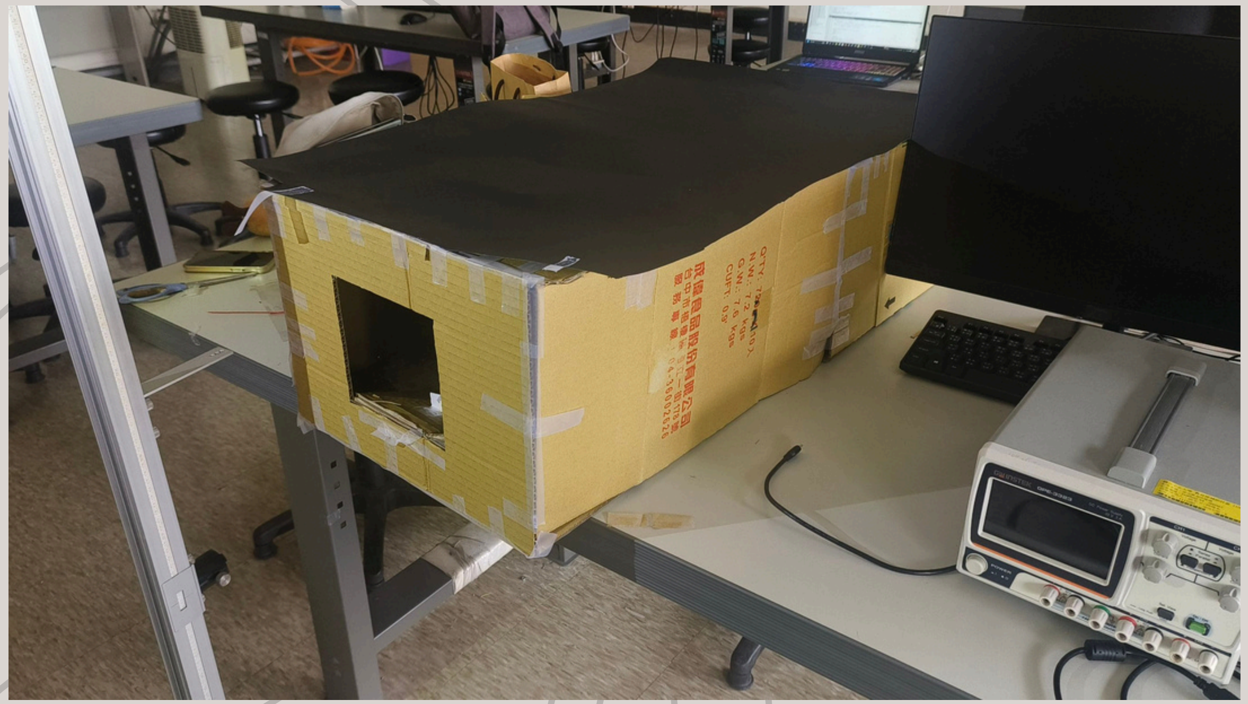
We can obtain the following chart.

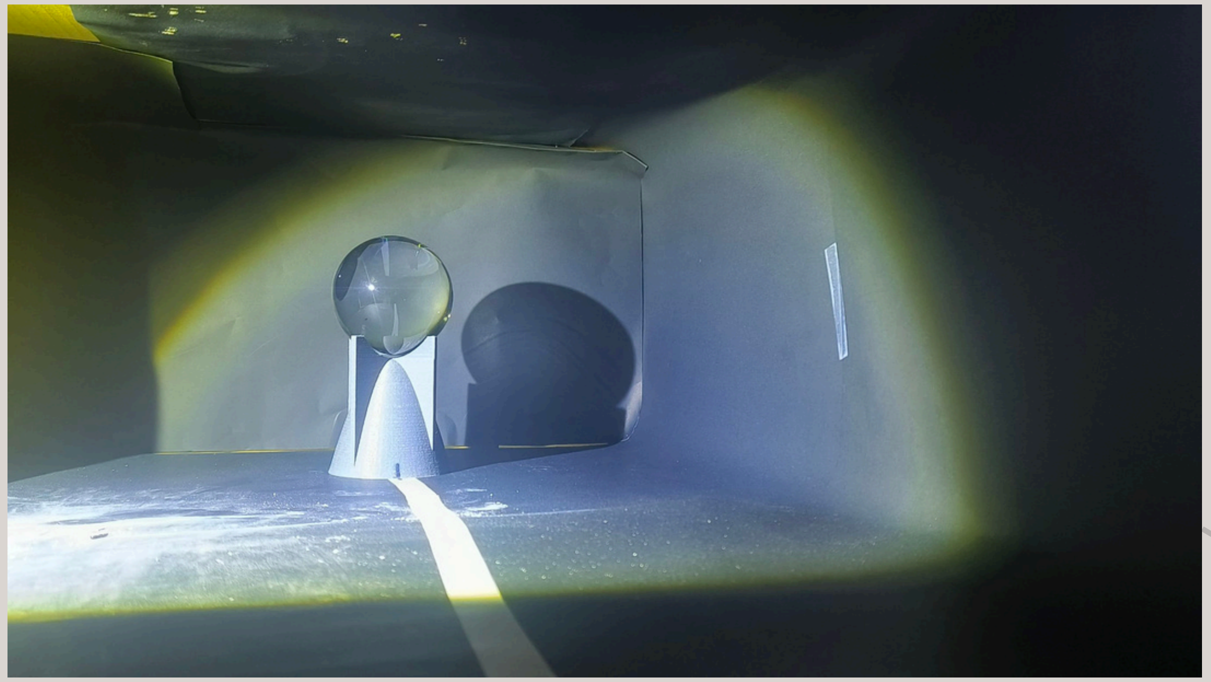


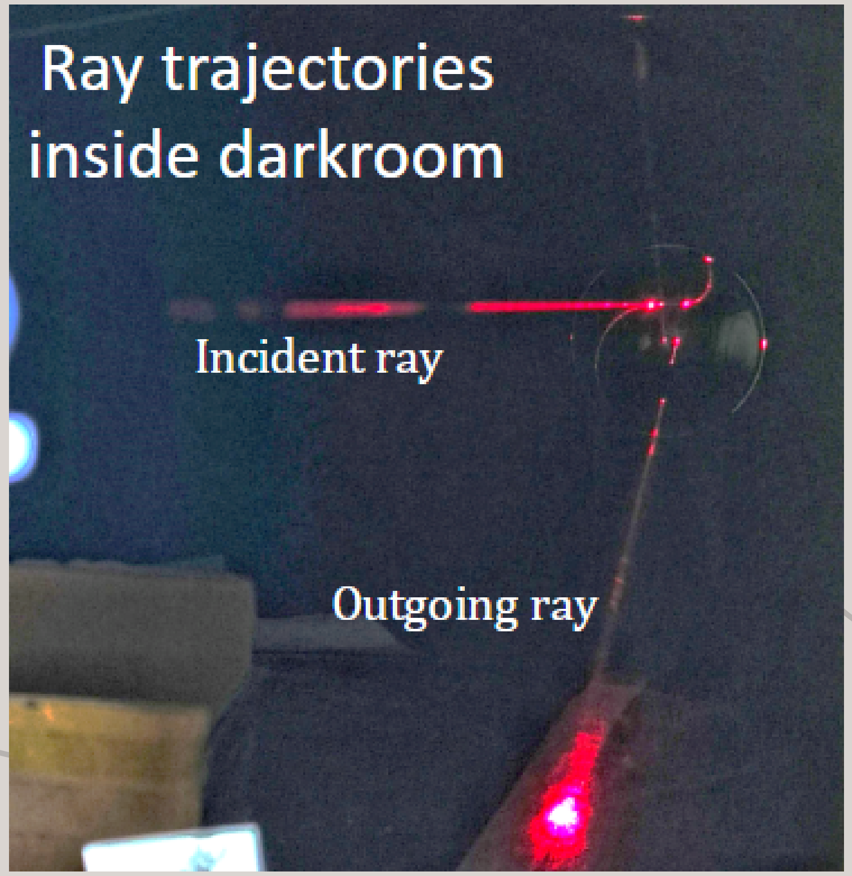
Light with a longer wavelength will have a larger viewing angle.

Experimental setup

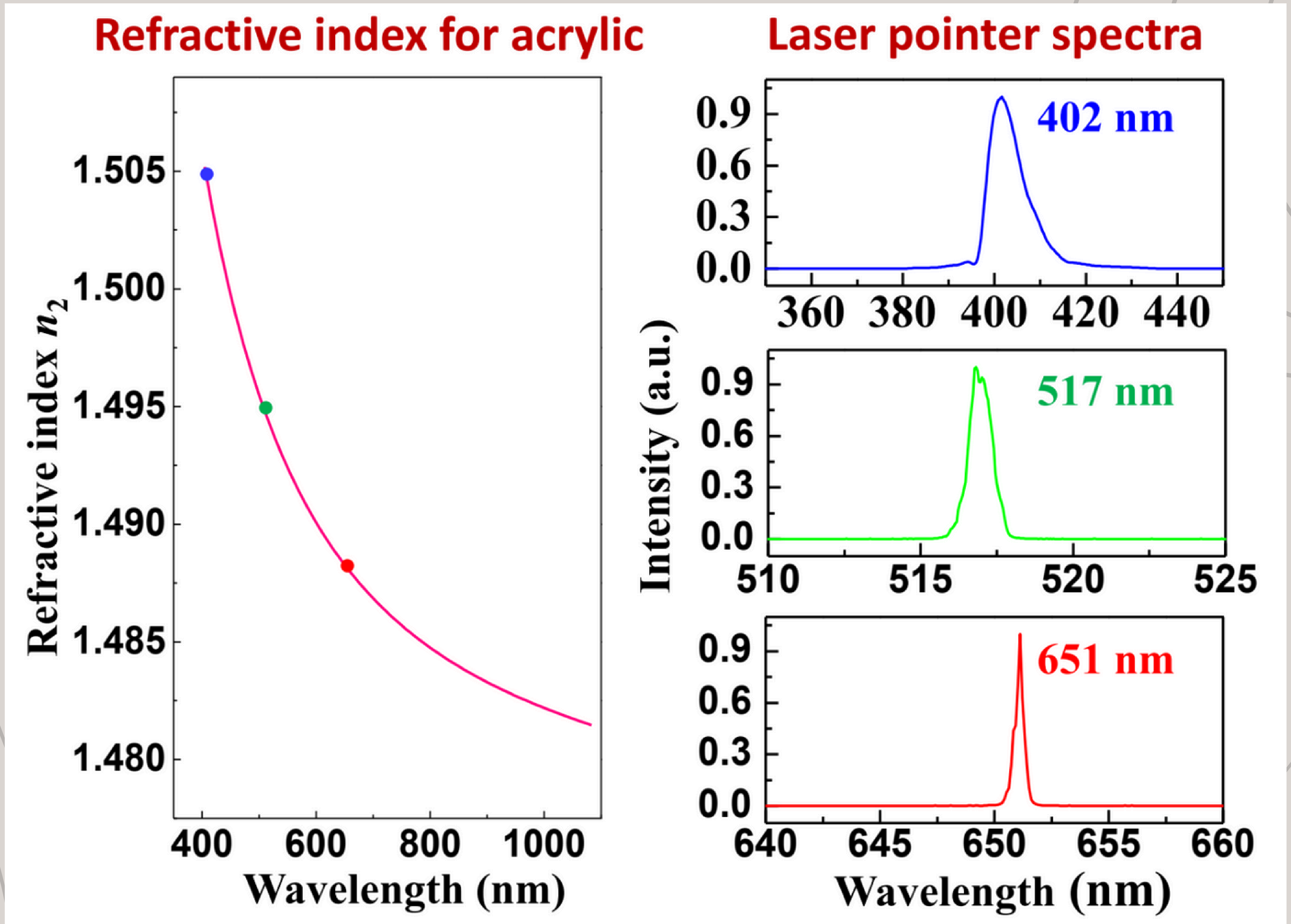




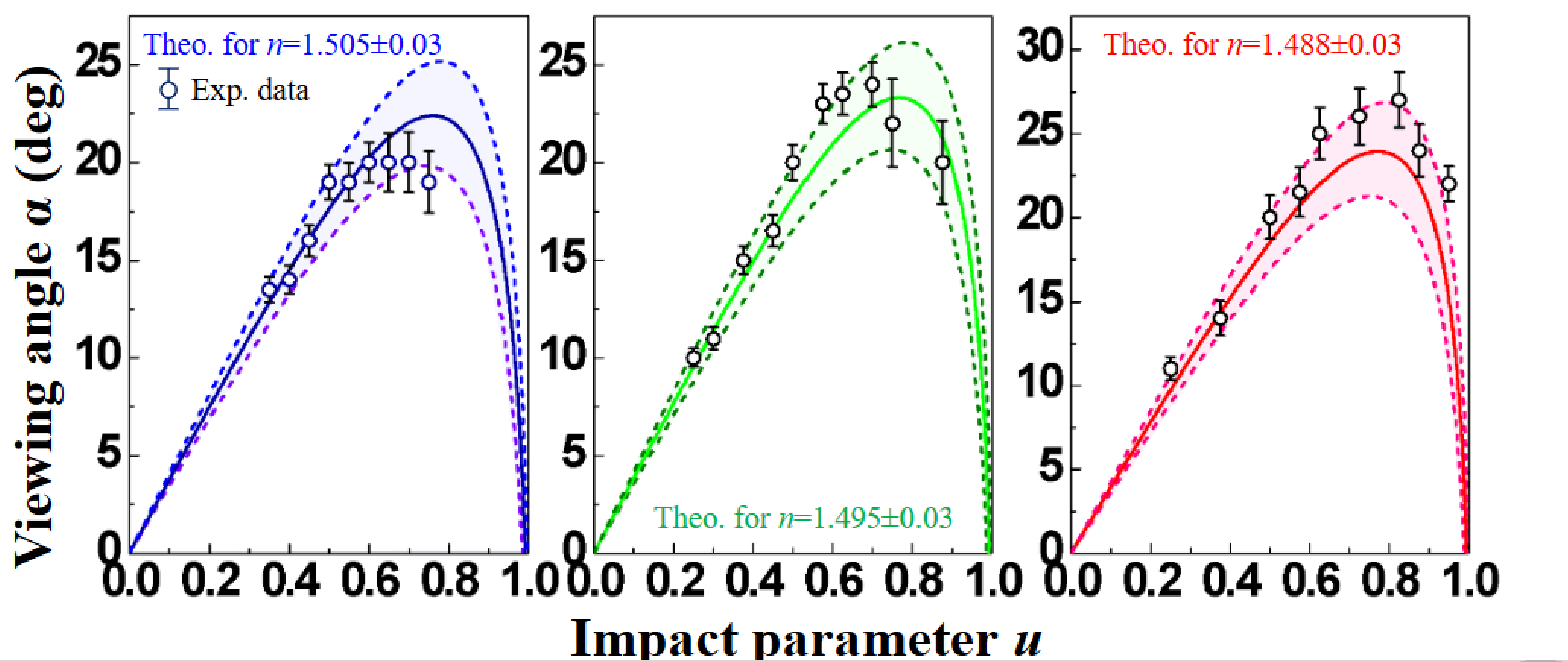




Experimental setup



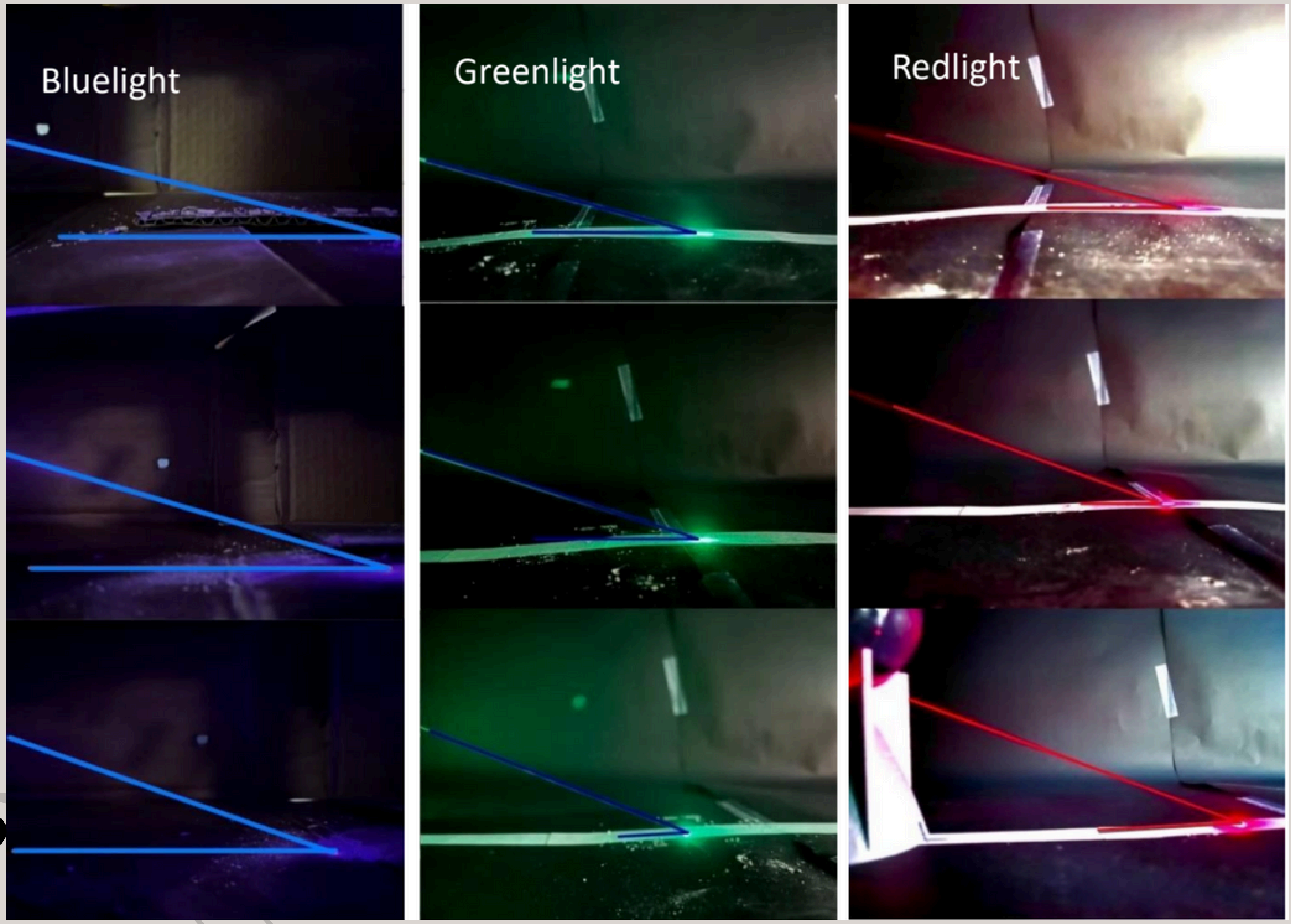
Result

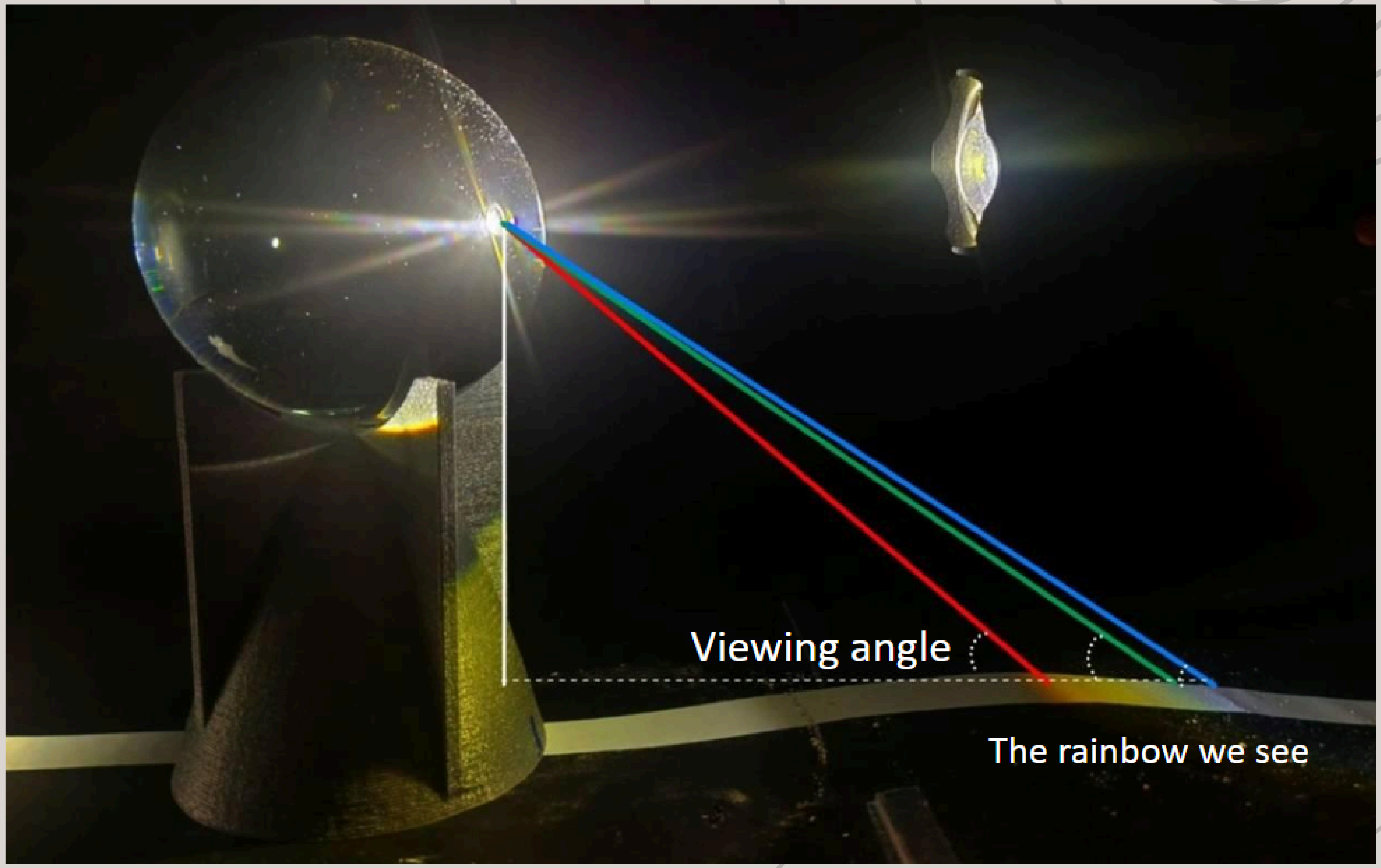


Substituting the corresponding refractive indices into the formula, α versus u and αmax for different color rays can be calculated to compare with the Exp. results.

Result

Increasing u





How does the light intensity vary at the maximum angle where the rainbow is observed, given that the viewing angle of rainbow formation depends on the ratio between the incident height and the radius of the raindrop?

Theoretical Principle

Consider the intensity relationship of light, as a function of height, after one reflection and two refractions.The following relationship can be obtained:

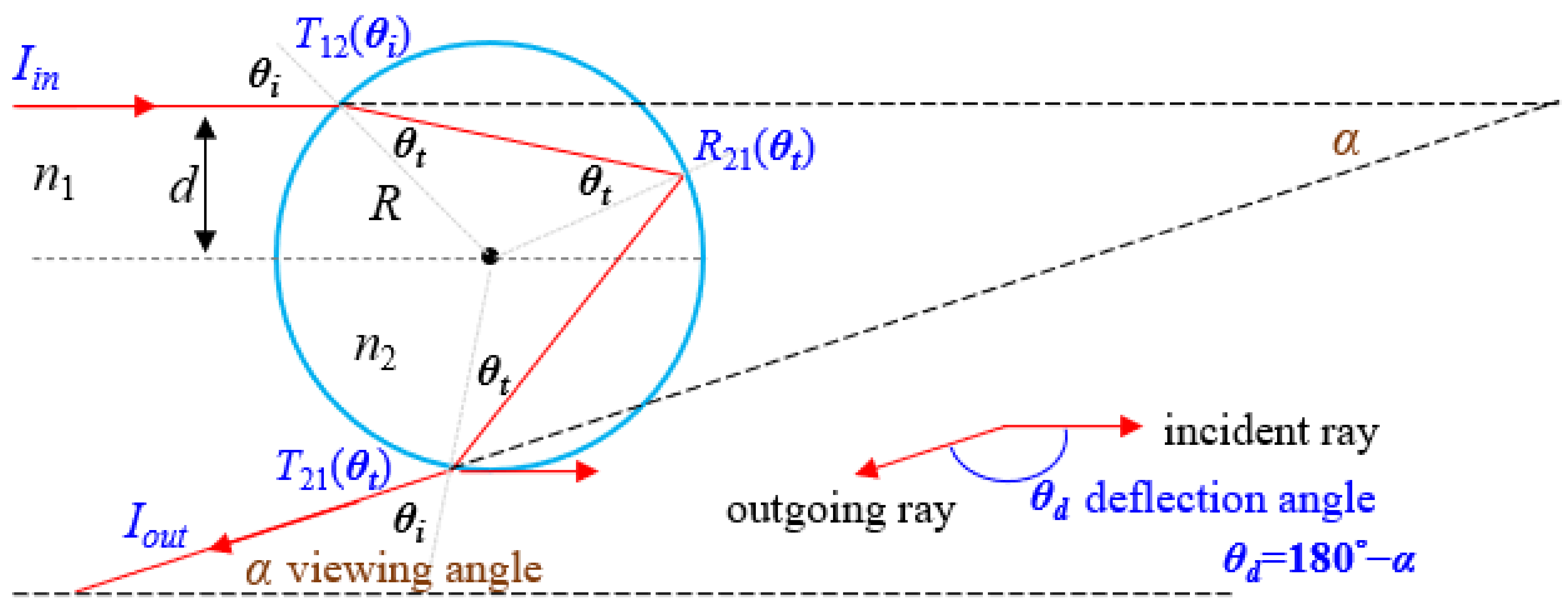
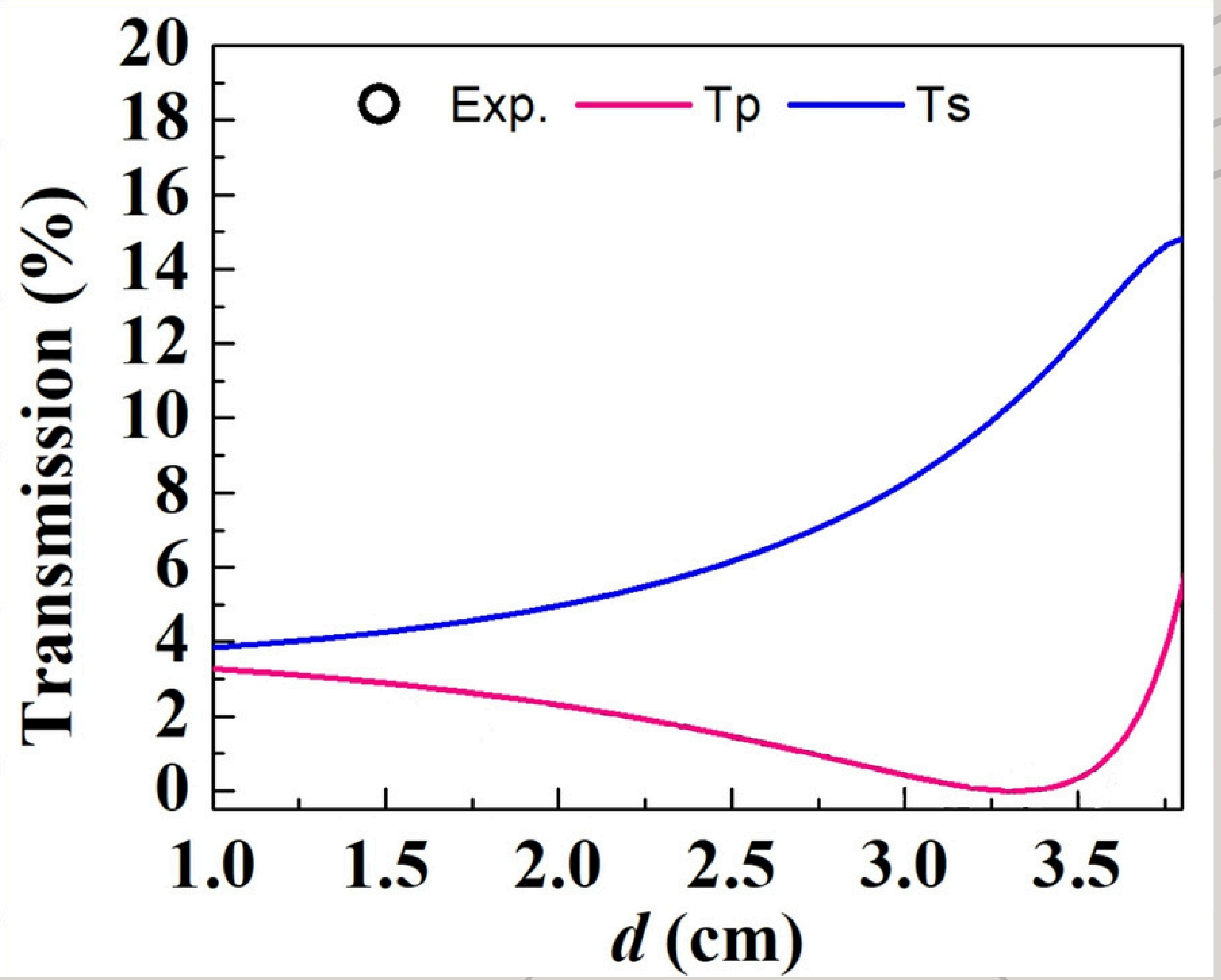


Fig. 1

$$I_{out}=I_{in}\cdot T_{12}(\theta_i)\cdot R_{21}(\theta_t)\cdot T_{21}(\theta_t)$$

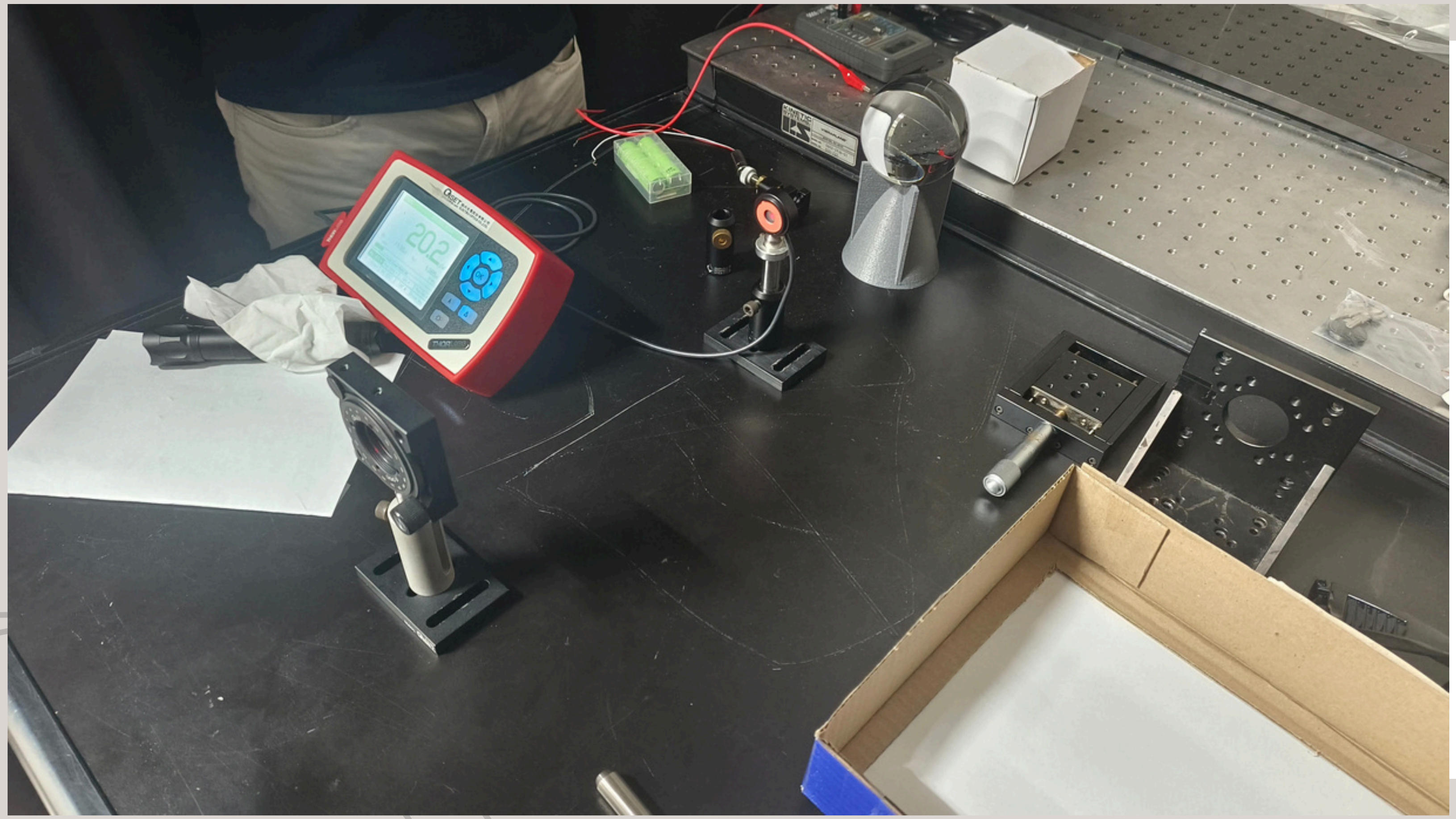


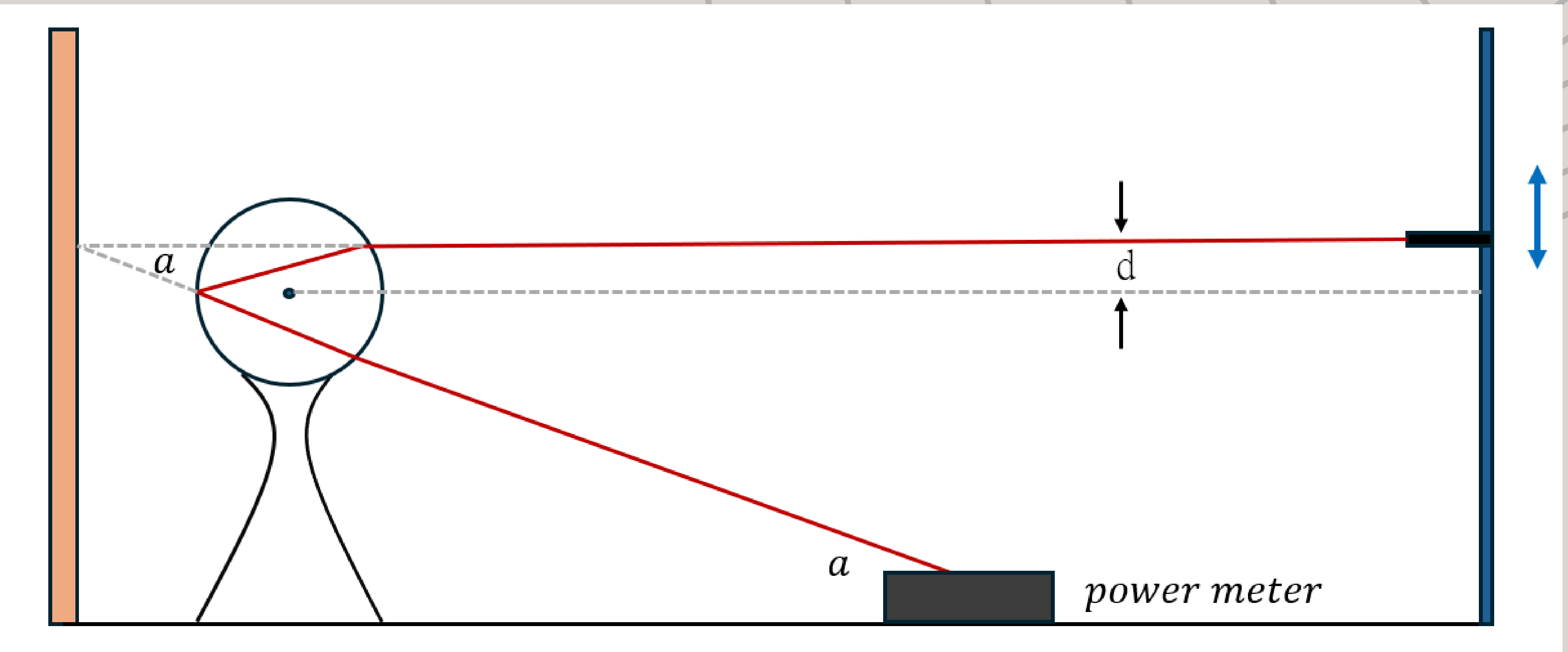
Without considering absorption, the transmittance becomes:

$$r_s(\theta)=\frac{\cos\theta-\sqrt{n^2-\sin^2\theta}}{\cos\theta+\sqrt{n^2-\sin^2\theta}};\quad r_p=\frac{n^2\cos\theta-\sqrt{n^2-\sin^2\theta}}{n^2\cos\theta+\sqrt{n^2-\sin^2\theta}};\quad n=n_2/n_1$$

$$\begin{cases} R_{s,p}(\theta)=\left[r_{s,p}(\theta)\right]^2 \\ T_{s,p}(\theta)=1-R_{s,p}(\theta) \end{cases}$$

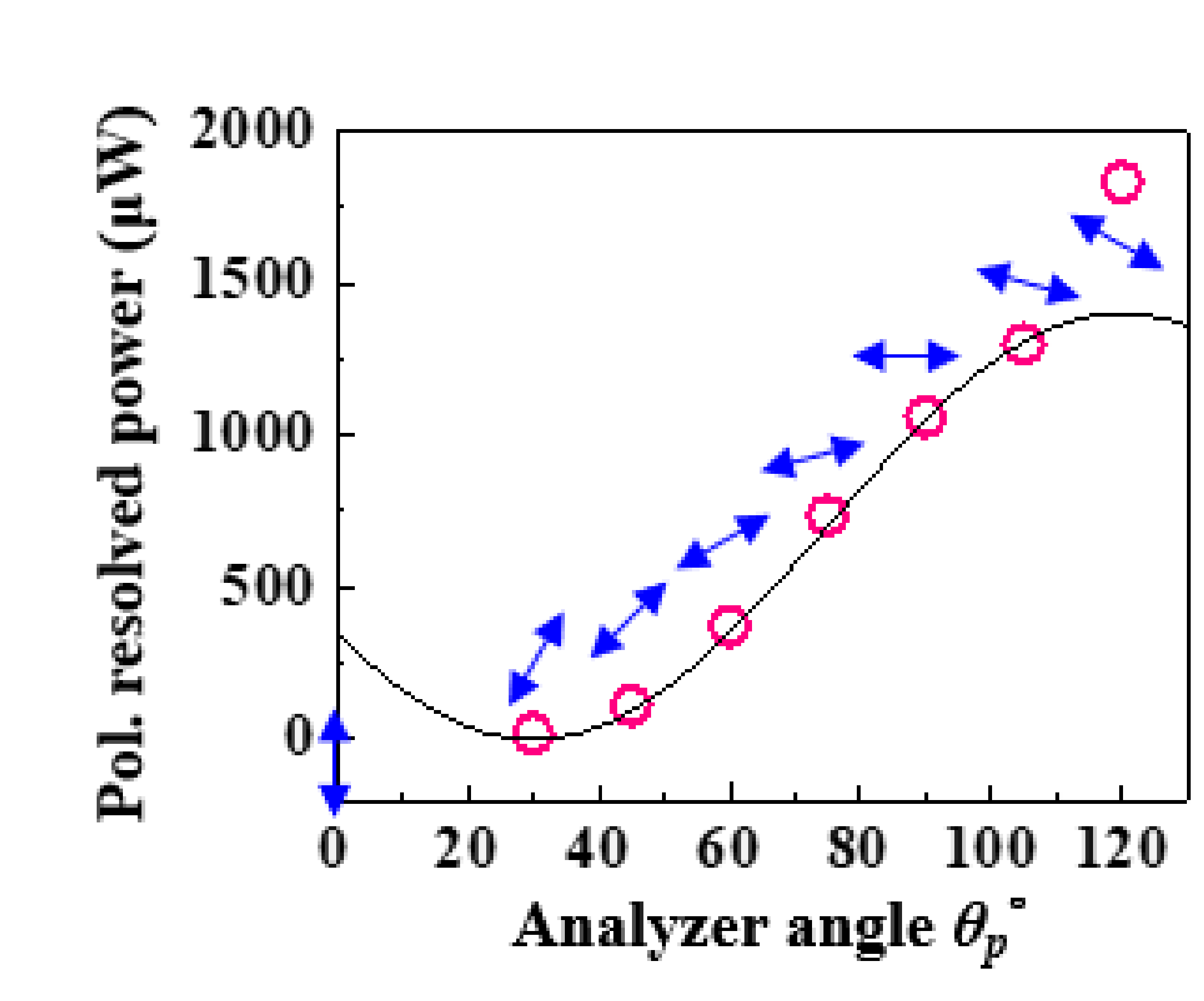
Experimental setup





The intensity of the un-refracted red light is measured to be 2.2 mW.

Result



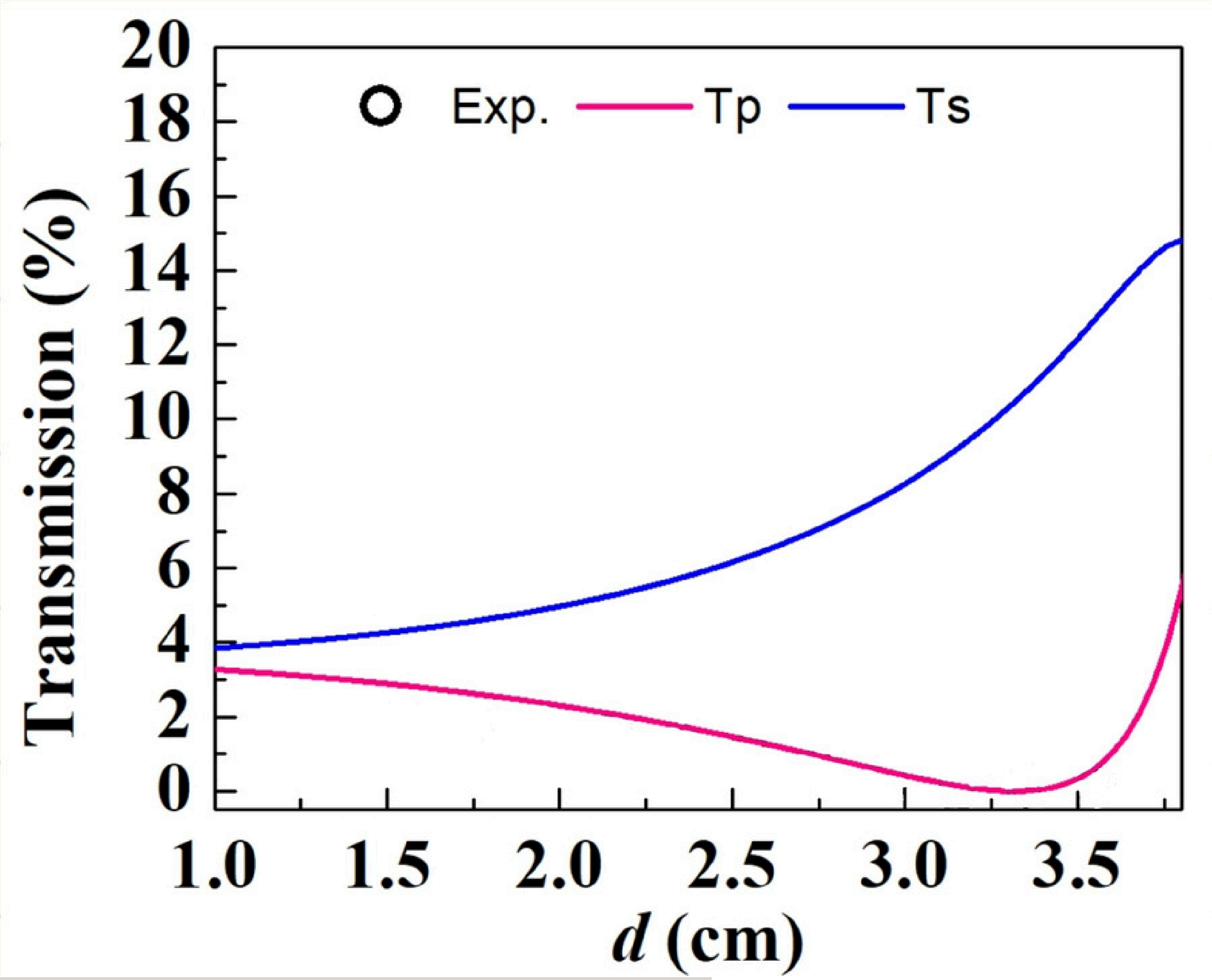
According to Malus's law:

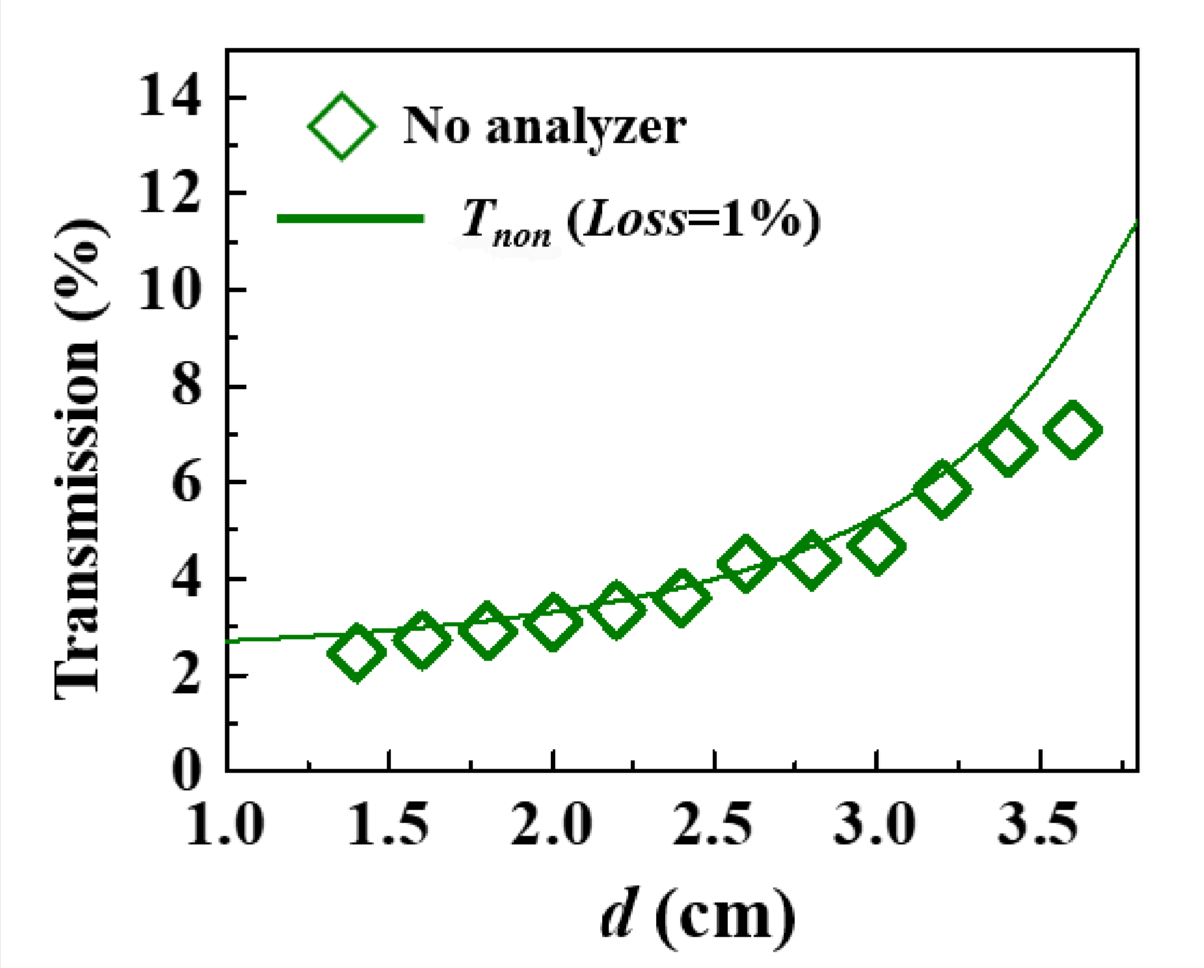
$$I_p(\theta_p) = I_o \cos^2\left(\theta_p - \theta_o\right)$$

The diode laser for the pointer is elliptically polarized with the polarization orientation along θo=120° relative to the vertical axis.

Result

By superimposing the theoretical value with the obtained polarization axis angle and subtracting the possible absorption error, the above equation is obtained.





$$I_{out} = I_{in} \cdot T_{12}(\theta_i) \cdot R_{21}(\theta_t) \cdot T_{21}(\theta_t)$$

$$T_{non} = T_{S-wave} \sin^2 \theta_o + T_{P-wave} \cos^2 \theta_o - Loss$$

As the height increases, the emitted light intensity also gradually increases.

Conclusion

α increases with u, reaches a maximum, then decreases rapidly.

At αmax : rays undergo 2-time refraction and 1-time TIR into the same outgoing direction.

while the maximum elevation angle and the intensity may not be directly related.

Considering type B uncertainty, the height is 0.29 cm,
the power meter reading is 0.29 μW, and the angle is 0.29 degrees.

Thank You