National Chung Cheng University,
Chiayi, Taiwan.

Quantitatively Studying Temperature-Dependent Sound Speed in an Air Column

Author：Chen-Han Tsai (蔡承翰), Pi-Hui Tuan (段必輝)

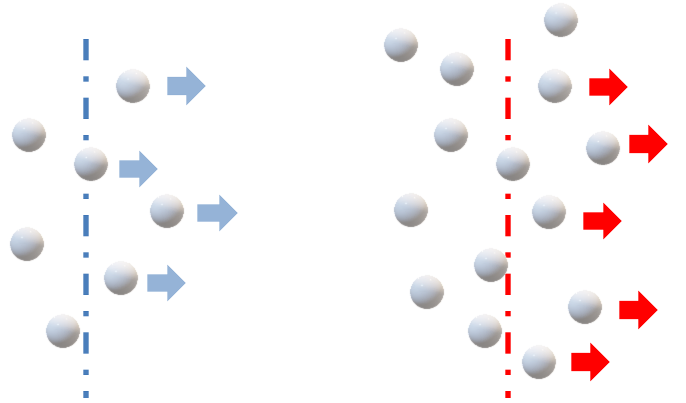
TABLE OF CONTENTS

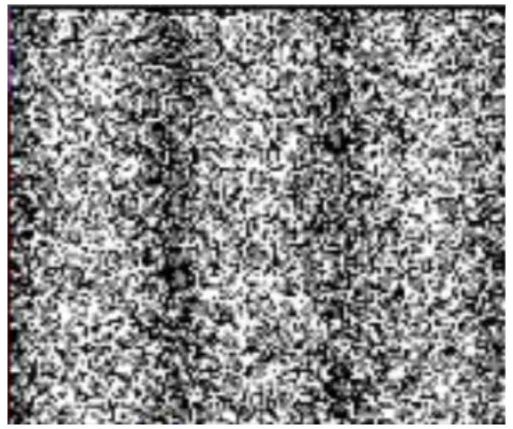
Abstract

This work aims to quantitatively verify the dependence of sound speed on the temperature in an air column. After deliberately calibrating the time-of-flight measurement between the transmitting and receiving Arduino-based ultrasonic transducers, the experimental result agree well with the theoretical prediction to show a good linear relationship to the increasing air temperature.

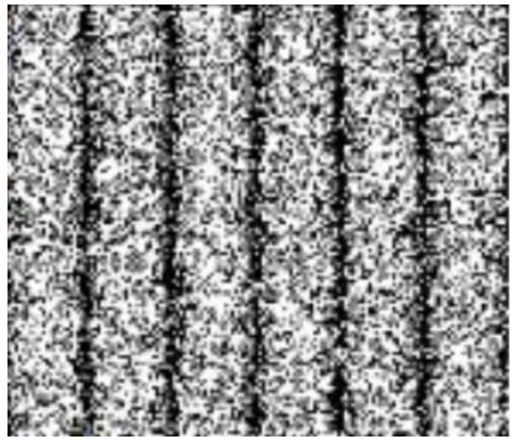
Next, we aim to utilize the precision of sensors to measure the variations in sound velocity and propagation time within a nonlinear temperature layer. By analyzing the propagation time difference, we can infer the temperatures at both ends of the nonlinear region, thereby developing a method that enables non-contact prediction of the temperature at the opposite end.

Theoretical principle



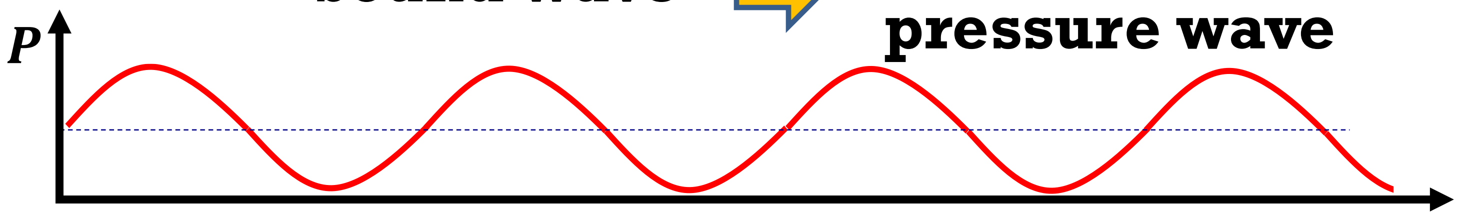


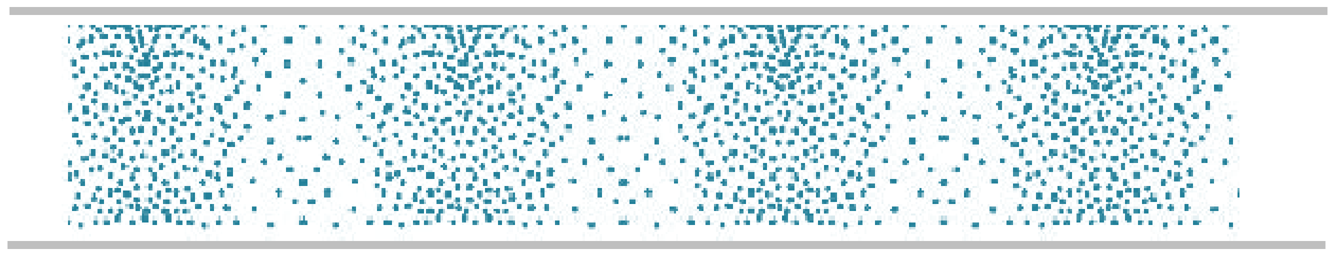
Low Temperature



high Temperature

Sound wave ⇒ **Longitudinal pressure wave**





Air molecule distribution

γ	Adiabatic index	**~1.4**
R	Specific heat of air	**8.314 J/mol·K**
M	Molar mass of air	**~0.029 kg/mol**

Sound speed in an ideal gas

$$v = \sqrt{\gamma RT/M}$$

@ 0˚C $v_{0°C} \cong 331.3$ (m/s)

@ *T*˚C $v_{T°C} \cong 331 + 0.6T°C$ (m/s)

Calculating the speed of sound at 0˚C by substituting the air parameters:

$T = 273.15$ K -Ideal gas constant

$M = 0.029$ kg/mol - mass of the gas

$R = 8.314$ J/(mol · K) -Specific heat of air

$$v_0 = \sqrt{\frac{1.4 \cdot 8.314 \cdot 273.15}{0.029}} \approx \sqrt{\frac{3177.8}{0.029}} \approx \sqrt{109579} \approx 331.3 \text{ m/s}$$

This is precisely the theoretical value of the speed of sound in air at 0˚C.

$$v = \sqrt{\frac{\gamma R}{M} \cdot (T°C + 273.15)} \qquad A = \sqrt{\frac{\gamma R}{M}} \approx 20.05$$

A first-order Taylor expansion around 0˚C (centered at 273.15 K) is performed:

Use: $v(T°C) = A \cdot \sqrt{T°C + 273.15}$

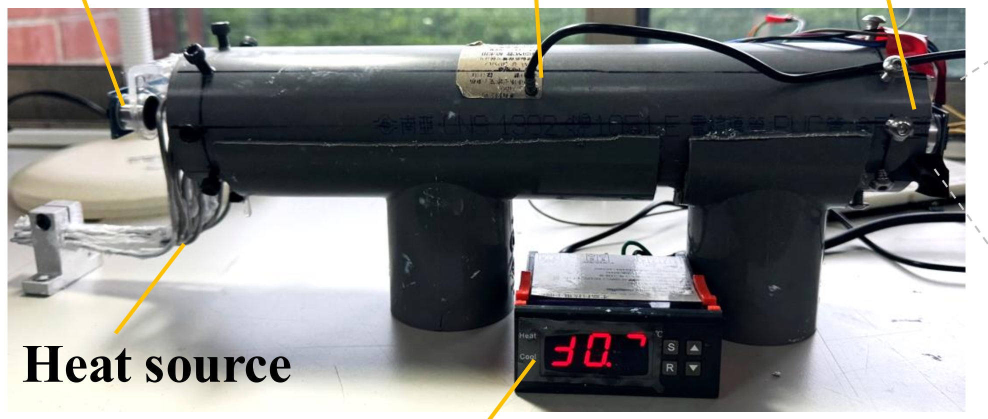
$$\sqrt{T°C + 273.15} \approx \sqrt{273.15} + \frac{1}{2\sqrt{273.15}} \cdot T°C$$

$$\sqrt{273.15} \approx 16.52, \quad \frac{1}{2 \cdot 16.52} \approx 0.0303$$

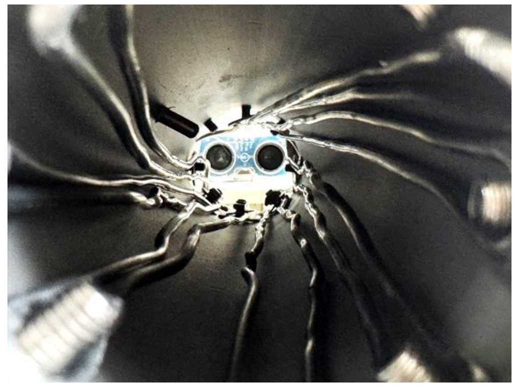
Therefore,

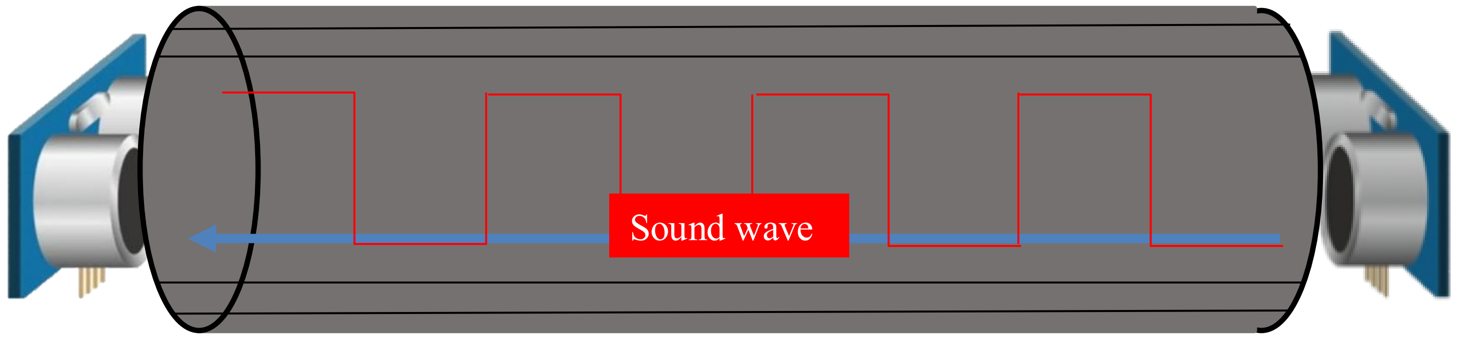
$$v \approx A \cdot (16.52 + 0.0303T°C) = 331 + 0.6T°C$$

uniformly heated air column (Gen 1)



Interior of the heated chamber





HC-SR04-R
(Acoustic receiving point)

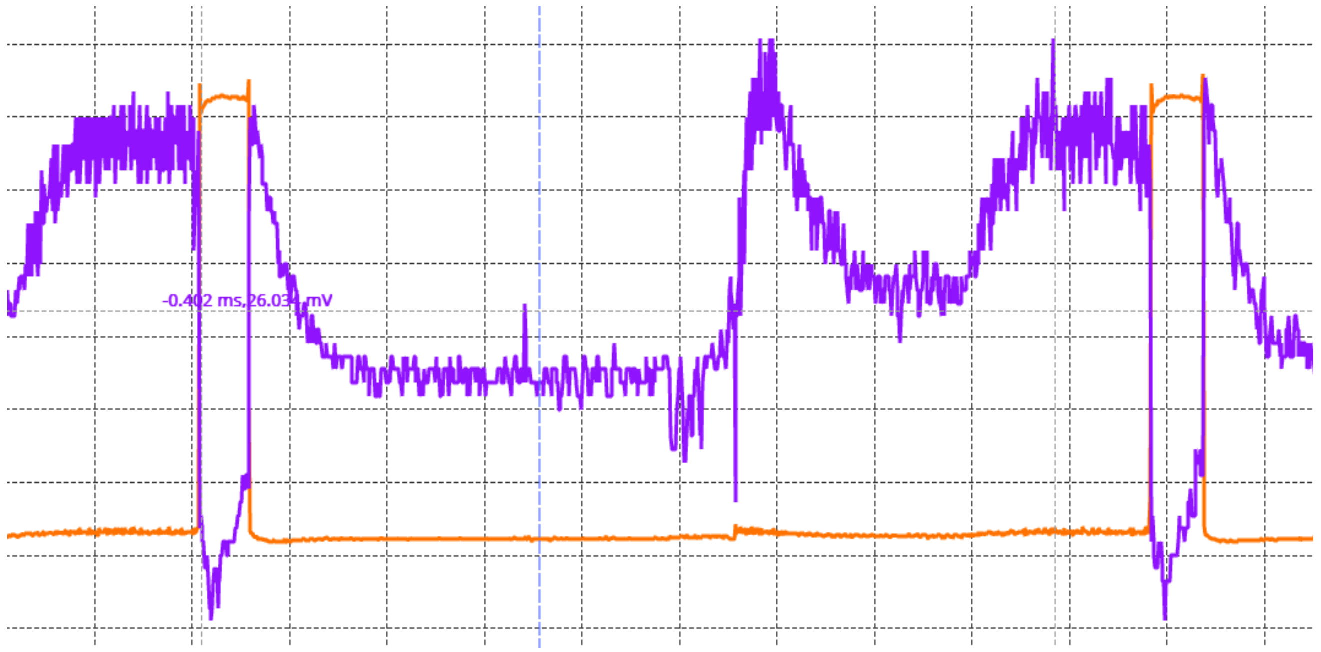
Connect Echo signal to CH2

HC-SR04-T
(Acoustic emission point)

Connect Trig signal to CH1

ADALM2000

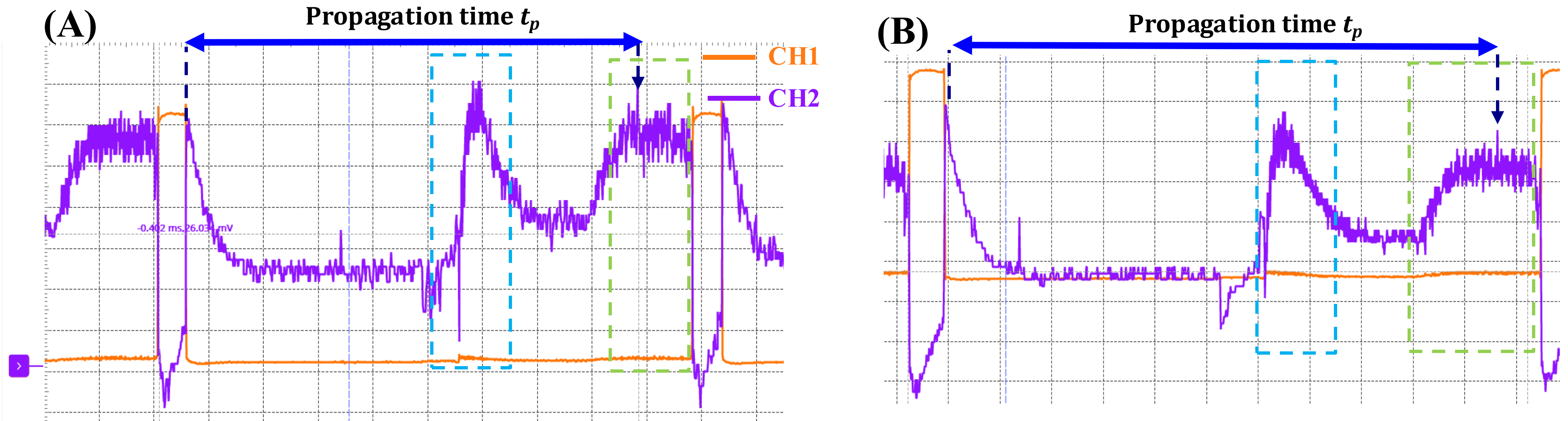
Function generator
Oscilloscope



We derive the sensing time from voltage waves.

The HC-SR04-T module was used as the transmitting device, emitting a 40 kHz square wave (chosen for its minimal interference). Another HC-SR04-T module served as the receiver. To avoid direct acoustic interference (crosstalk), the experimental setup was designed for unidirectional reception. Additionally, silver wires were applied at the heating end to increase the thermal conduction area, thereby ensuring a higher accuracy of the internal chamber temperature.

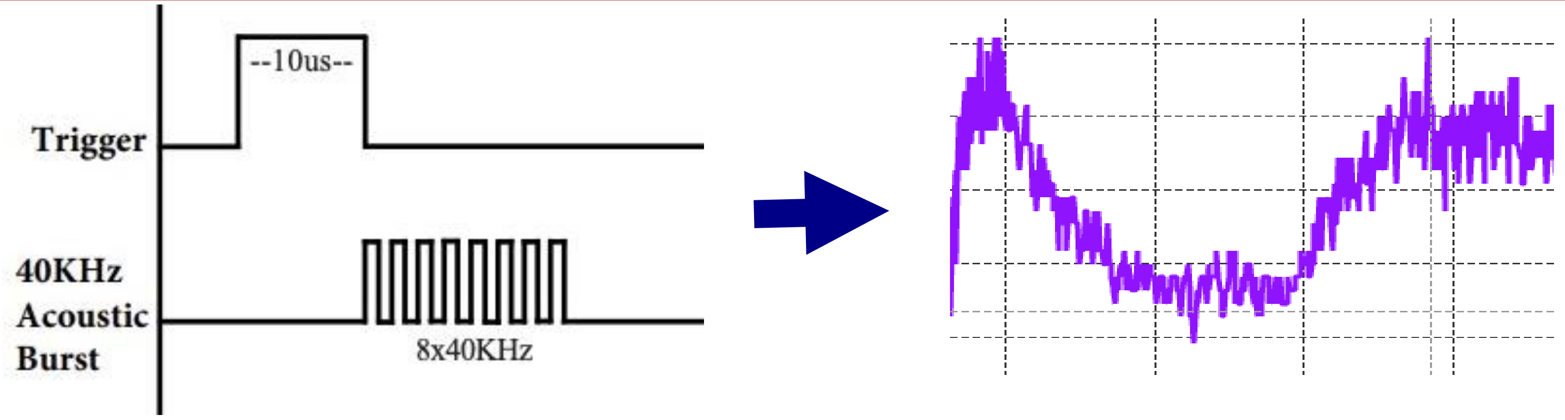
Time-of-flight measurement calibration



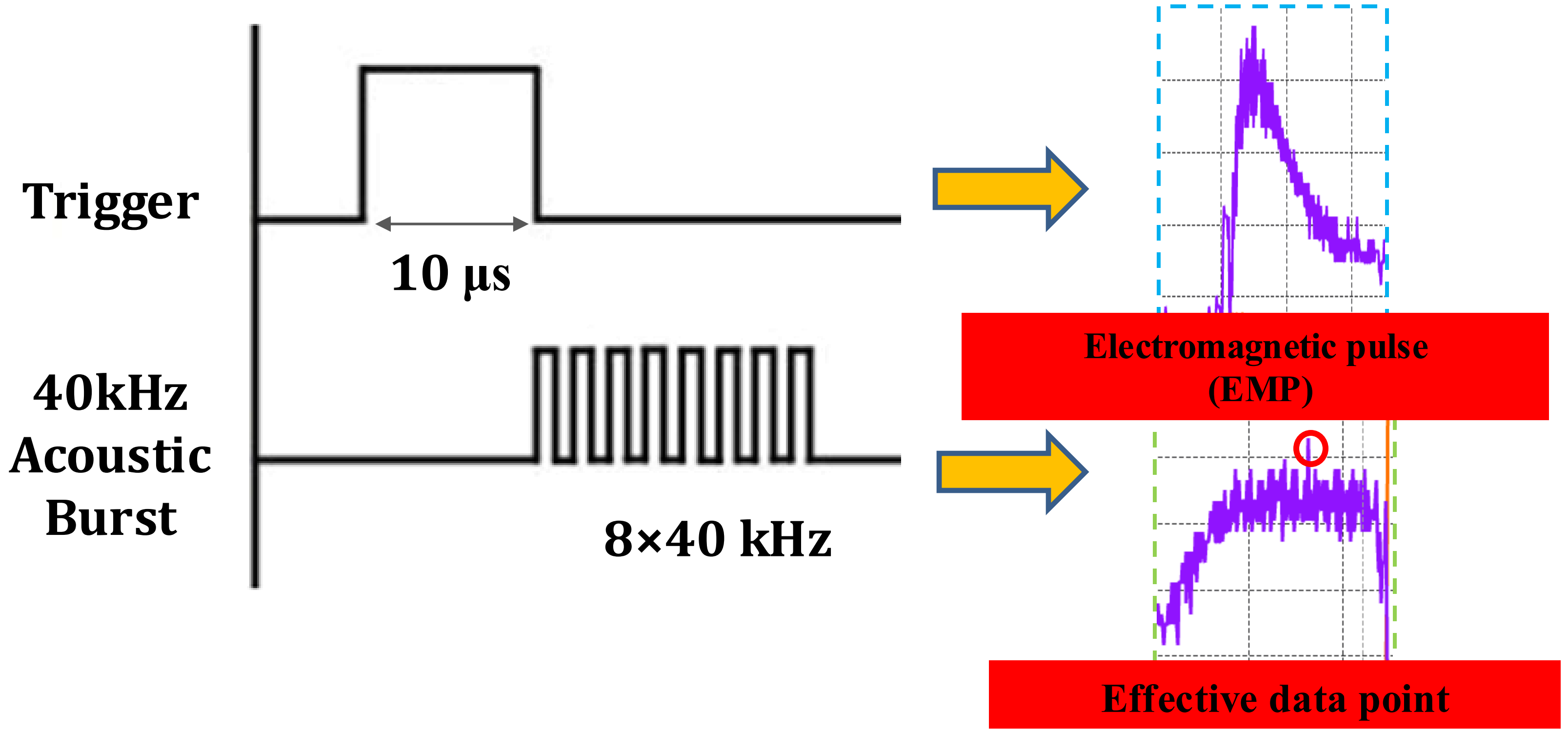
The above Figs. (A) and (B) show the temperature inside and outside the tube at T = 30°C.The results confirm that the measurement method is robust and not affected by external interference.

In the following analysis, we address the rationale for selecting the highest peak in the second segment as the measurement point. Additionally, we investigate the cause of the leading envelop, the underlying factors contributing to its presence, and the reason for the high-density waveform.

First, we explain the reason for the denser waveform based on the design and output characteristics of the HC-SR04 module:



The internal trigger in Scopy does not exactly match the actual output, as the output consists of multiple overlapping pulses. This results in greater waveform jitter.



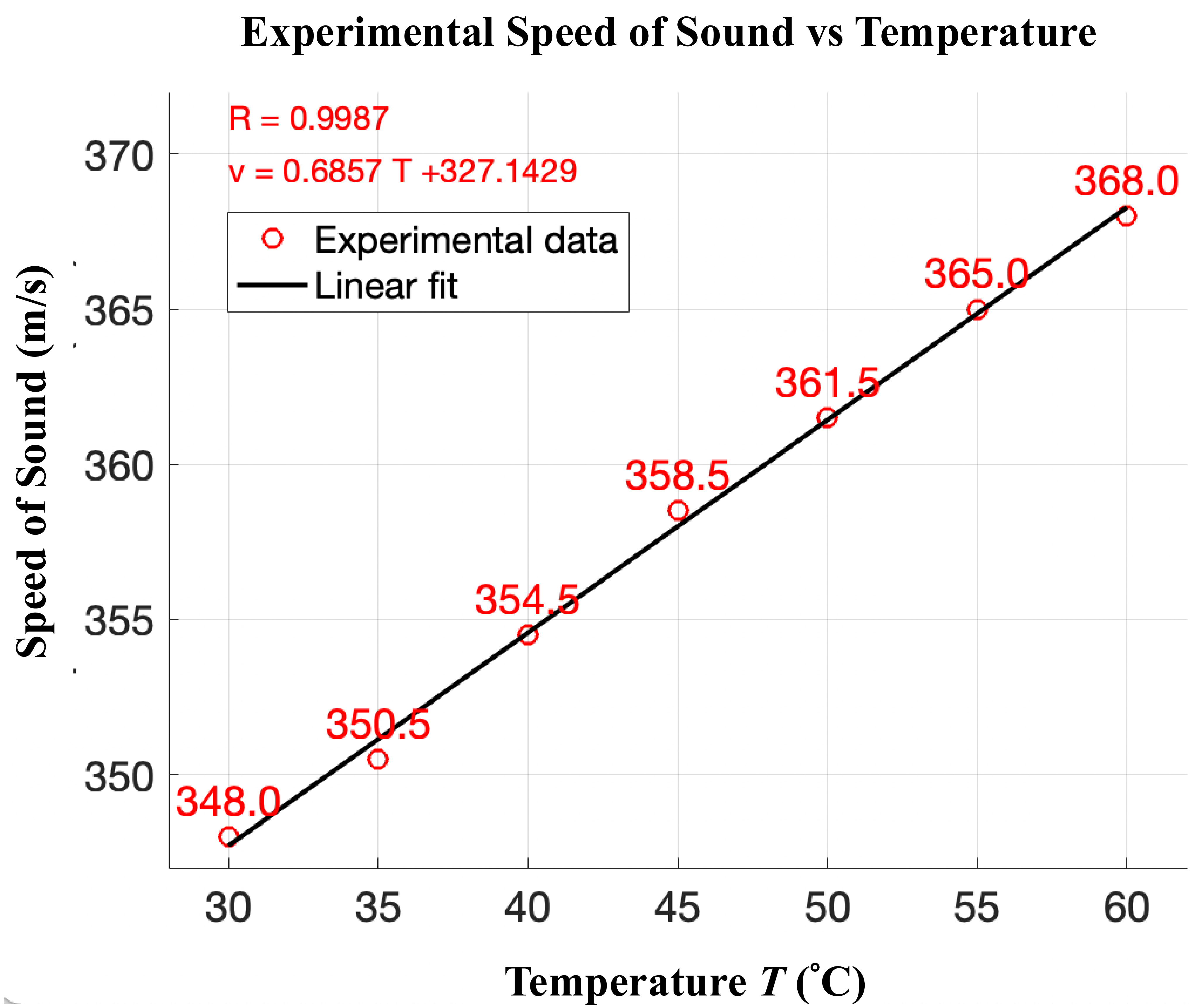
Leading envelop → EMP (Electromagnetic Pulse)

- Caused by the trigger voltage difference to induce the Faraday effect

Secondary envelop → Real sound ToF signal

- Caused by recieving **multiple 40-kHz bursts** from the HC-SR04 sensor.
- We choose the most prominent peak as the **effective data point.**

Results for uniformly heated air column



A linear fit of the measured sound speed versus temperature yields a slope of **0.6857**.

v_s=327+**0.685**T (m/s) —**Measurement**

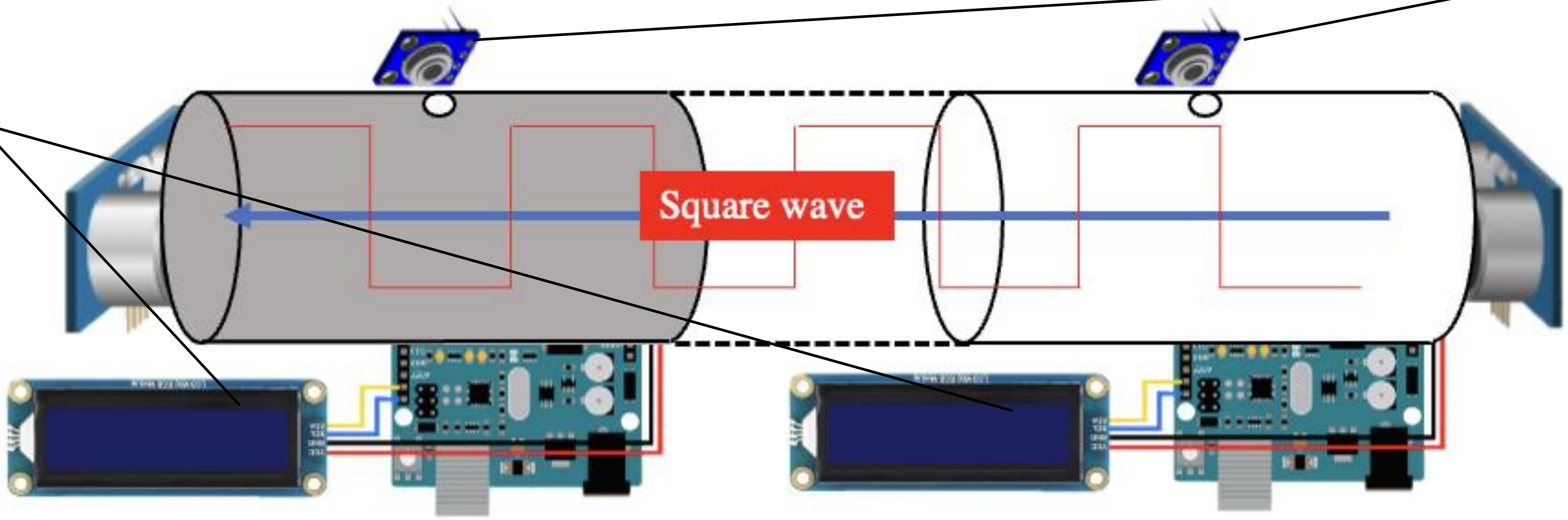
v_s=331+**0.606** T(m/s) —**Theory**

The errors for the measured sound speed at 30 and 40˚C to the theoretical values are only 0.4% and 0.3%.

At this stage, the device's accuracy and stability are verified.

Setup for air columns with ΔT (Gen 2)

-Measurement method

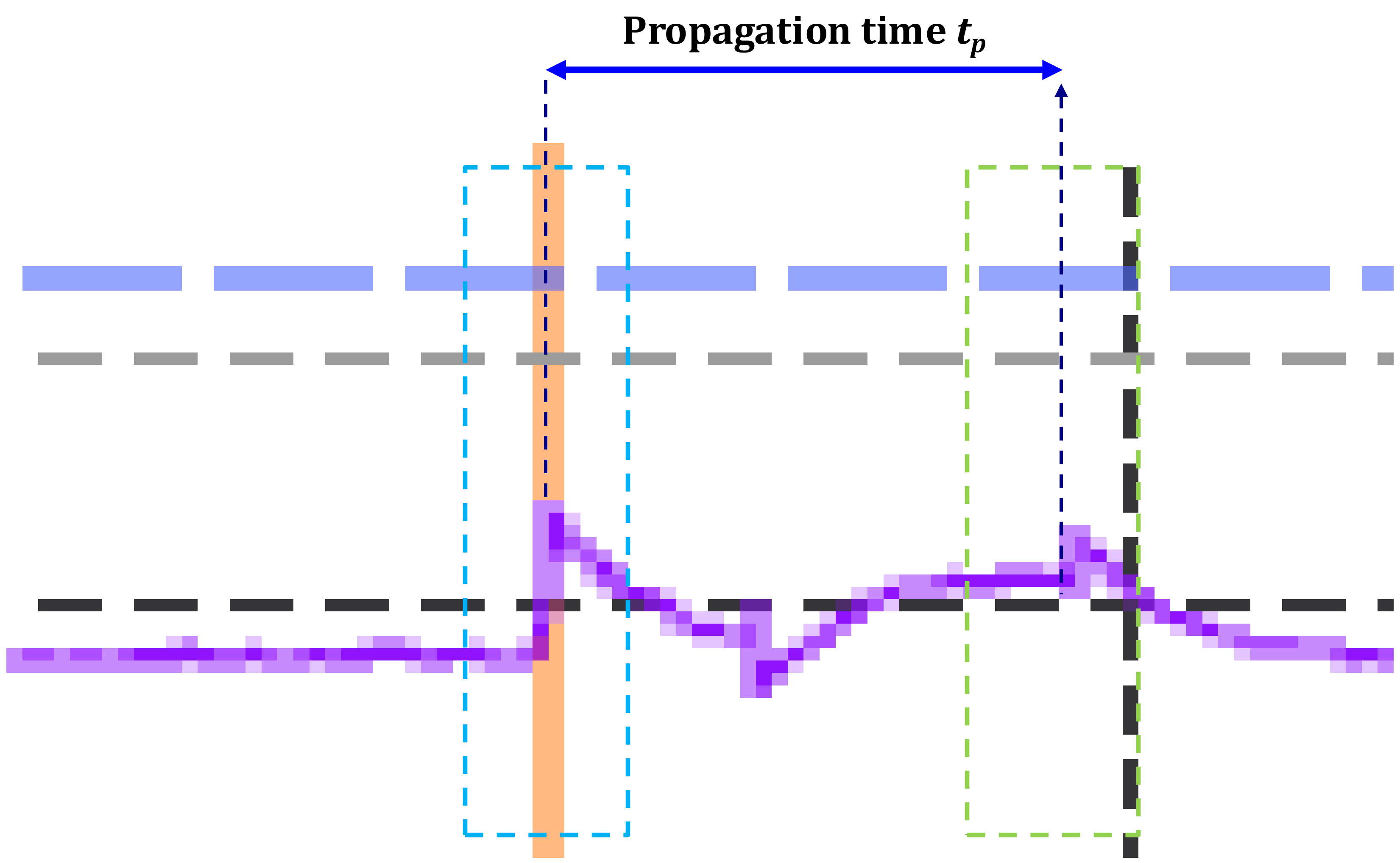


In the nonlinear temperature chamber, the original temperature controller was replaced with an MLX90614 infrared temperature sensor. By integrating it with an Arduino and a 1602 LCD display, the internal temperature of the chamber can be monitored directly. Although acrylic tubing was used for the chamber, the heating section was wrapped with aluminum foil to enhance heating efficiency and accelerate temperature rise.

Setup for air columns with Δ*T*

-Validation of Thermal Uniformity in the Heated Tube

Method: The actual measured values (without considering intermediate heat transfer) were compared with the experimentally obtained time data. We assume that small deviations fall within an acceptable error range, since terminal heat transfer effects are not taken into account.

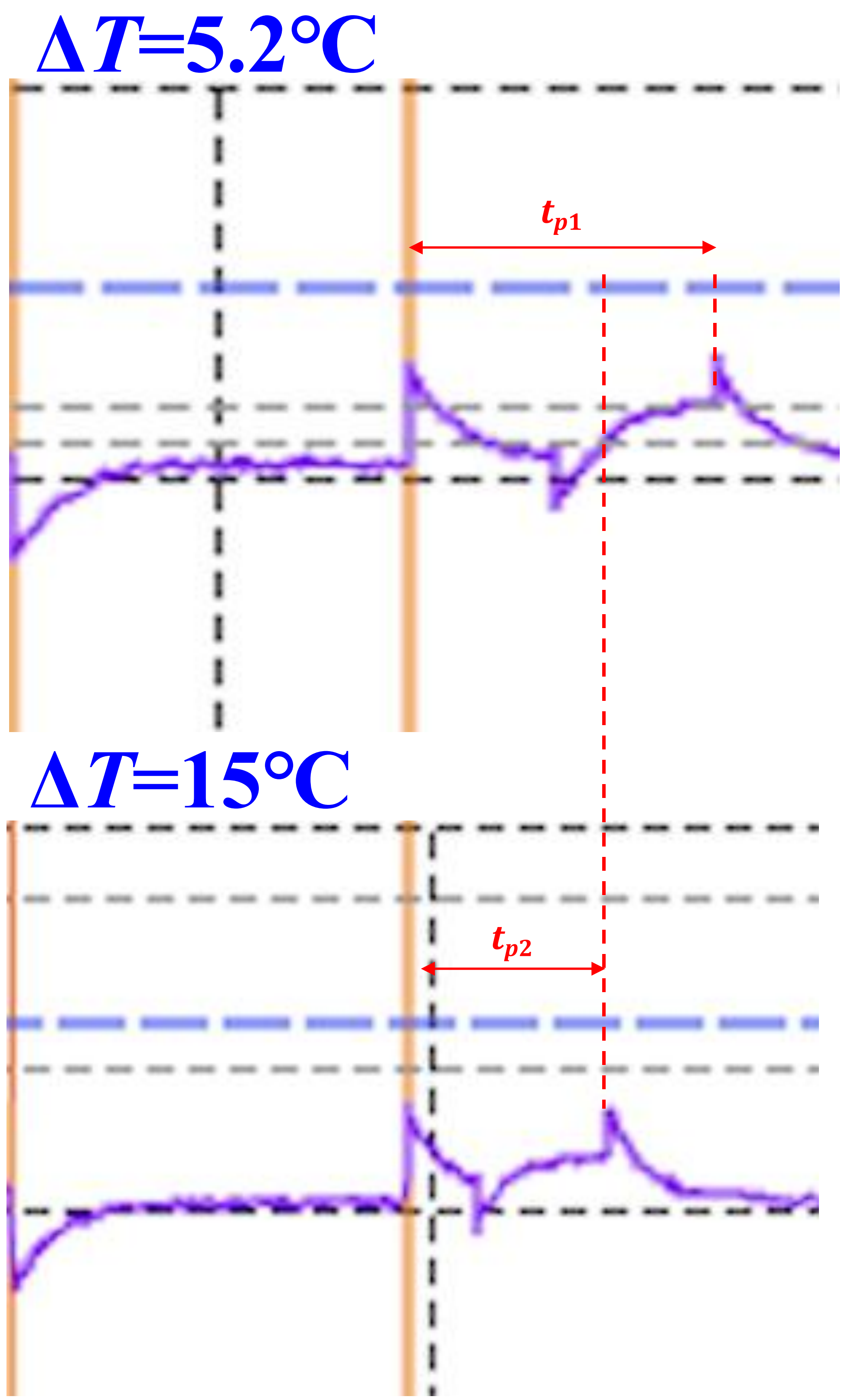


The theoretical sound speed inside the column under a uniform 30°C condition is **349.81 m/s**. According to the left Fig., the measured time t_p for traveling a 0.616-m distance is 171.3 μs, resulting in a calculated speed of **349.53 m/s**.

The observed error of only 0.3 m/s is attributed to heat conduction. Therefore, the methodology—including temperature control, equipment setup, and sampling procedures—was confirmed to be feasible. We then proceeded to discuss scenarios with different temperature gradients.

Setup for air columns with ΔT

-Signal Timing and Simplified Theoretical Analysis



Simplified model

Segment Temperatures

$$T_1 = 29.8^\circ\mathrm{C}, \quad T_2 = \text{variable}$$

Pipe Lengths

$$L_1 = L_2 = 0.308\,\mathrm{m}$$

Travel time in each segment

$$t_1 = \frac{L_1}{v(T_1)} \quad ; \quad t_2 = \frac{L_2}{v(T_2)}$$

Total propagation time

$$t_{\text{theo}} = t_1 + t_2 = \frac{L_1}{v(T_1)} + \frac{L_2}{v(T_2)}$$

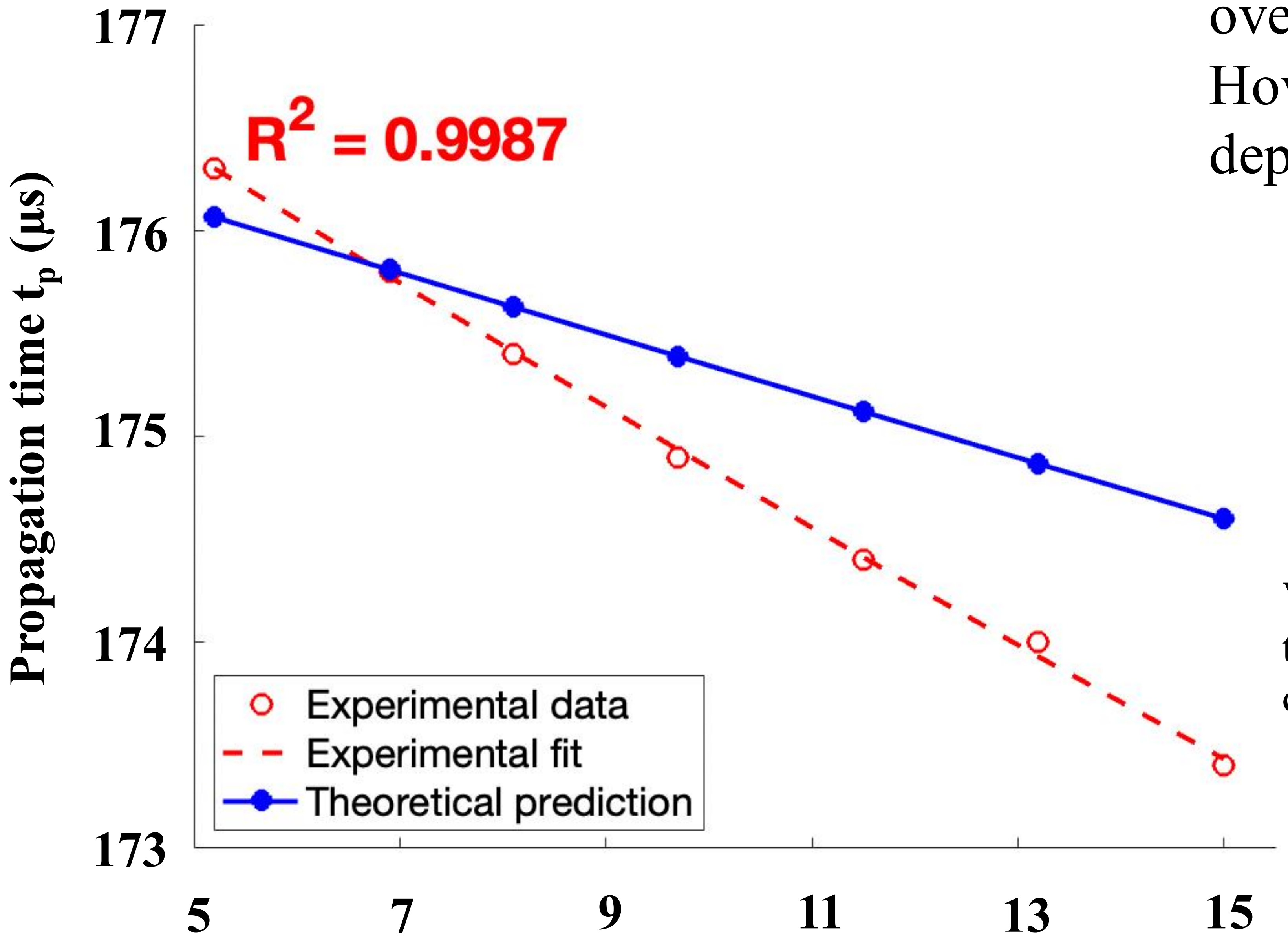
The measurement result (upper-left Fig.) shows that as ΔT increases the propagation time t_p decreases. This agrees well with the theoretical prediction by the simplified model shown at upper-right part.

Setup for air columns with ΔT

-Function Inference

After completing all calibrations, we collected data along seven different temperature gradients for analysis. Finally, the data were fitted using FFT to derive a linear equation.

Comparison between Exp. and Theo. t_p



The deviation between the exp. and theo. t_p may come from the oversimplified model ignoring the thermal convection inside the column. However, the simplified model can still nicely reveal the negatively linear dependence of t_p on the ΔT.

Experimental fitted result:

$$t_p = 178.66 - 0.667\Delta T \text{ (μs)}$$

When measuring the temperature at one end, the temperature at the opposite end was inferred using the recorded time data. By determining the temperature difference, the corresponding time for the other end was estimated.

Conclusion

The fitting results agree well with the theoretical model to confirm a linear relationship between the temperature and sound speed. Even though there is deviation for the absolute measured travel time from the predicted values by the oversimplified model without considering the heat transfer, the home-made setup nicely demonstrates the temperature-dependent sound speed inside the air column for pedagogical purposes.

Reference

[1] Cheng-You Lee, Chih-Hsien Huang, “Sound speed estimation of environment with non-uniform gas temperature distribution.” MS Thesis, Dept. of Electrical Engineering, NCKU (2021).

[2] P. S. Carvalho *et. al.*, “Determining the Speed of Sound as a Function of Temperature Using Arduino”, Phys. Teach. **57**, 114-115 (2019).